Why Take This Module?

The topic of reservoir fluid displacement is important, since one can set up a fractional flow equation for a new reservoir before a complicated model, and get upper end recovery factors:

\[
f_w = 1 - \frac{0.001127 \: k_o \: \mu_w q_i}{\mu_w} \left[ \frac{\partial p_w}{\partial u} + 0.434 (\Delta \gamma) \sin \alpha \right]
\]

\[
1 + k_o \cdot \frac{\mu_w}{\mu_o}
\]

A frequent problem a reservoir engineer faces: “What is the upper bounding limit of recovery factor?”

Why Take This Module?

- Doing a fractional flow curve can help set the upper bounding limit
- The use of a simple fractional flow curve, as covered in this model, can be used to prevent significant investment losses
Reservoir Fluid Displacement Core

Dispersed Flow

Learning Objectives

This section will cover the following learning objectives:

- Recognize immiscible fluid displacement and sources of data used for calculations
- Describe the assumptions and limitations of the Buckley-Leverett theory
- Describe the derivation of the dispersed fractional flow equation
- Recognize the impact of mobility, gravity, and capillary pressure on the dispersed fractional flow equation
- Identify how the Welge solution can be applied before, during, and after breakthrough
In This Section

Immiscible Fluid Displacement

Where does immiscible fluid displacement occur?

Fractional Flow
- Capillary Pressure
- Fluid Mobility
- Gravity Term

Buckley-Leverett Theory

Frontal Advance and Welge Solution

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Where Does Immiscible Fluid Displacement Occur?

This section will cover immiscible fluid displacement. Immiscibility of two fluids refers to the fact that the displaced and displacing fluid do not mix under any concentrations at reservoir conditions. As you learned in another reservoir engineering module on capillary pressure, whenever you have two such fluids in a confined space (the pores of a reservoir rock) you have capillary pressure.

Immiscible injection occurs whenever there is a capillary pressure between the displaced and displacing phase. Some examples include:

- Natural water influx from an aquifer
- Gas-cap expansion during pressure depletion
- Secondary recovery operation
Where Does the Data Come From?

Types of Data

You will have covered the derivation of relative permeability and the log sources of your data in other modules.

- Relative permeability equipment – laboratory
- Petrophysical – well logs, structural elevations
- Geological iso-pach maps with well layout in reservoir
- Production and injection data

Unsteady State Determination

Here is where data comes from in a laboratory. Two types of three-phase relative permeability apparatus are shown for un-steady state determination, on the left X-Ray and on the right electrical determination of relative permeability.
Steady State Determination

This is an apparatus for steady state relative permeability determination. You will now take the information generated by this equipment and apply it to the displacement process of water displacing oil.
What is the Buckley-Leverett Theory?

1. Calculates pore level displacement efficiency

2. Assumes “piston-like” displacement and linear flow
   This module uses an oil-water system but analogous equations apply for a gas-oil system and a water displacing gas system, these will be covered in the fundamental module of this series.

3. Some moveable oil is bypassed, due to:
   - Viscosity contrast
   - Relative permeability
   - Capillary pressure

4. Immiscible displacement characterized as:
   - Uniformly dispersed
   - Segregated

Buckley-Leverett Theory

Linear Systems

- Buckley-Leverett only applies to linear systems
  - Most injection patterns and real formation displacement systems are not linear
- Problem B, however, will cover a linear displacement system
  - Sweep efficiency
  - Horizontal and non-horizontal reservoirs
Buckley-Leverett Theory

Assume that a stabilized flood front develops, including:

Front Boundary
- Rapid saturation change

Ahead of the Front
- Displaced oil bank

Behind the Front
- Two-phase flow of oil and water

Buckley-Leverett Theory Assumptions

Assumptions
1. Incompressible in-situ and injected fluids
2. Constant average system pressure
3. Steady state flow
   - Same reservoir flow out as in
Fractional Flow at Reservoir Conditions

Assuming water as the displacing phase:

\[ q_{in} = q_{out} = q_o + q_w = q_t \]

Where:
- \( q_o \) = oil flow rate per unit cross-sectional area, at any point along the core
- \( q_w \) = displacing phase (here \( \rightarrow \) water) flow rate per unit cross-sectional area, at the same point in the core as \( q_o \)
- \( q_t \) = total flow rate per unit cross-sectional area, at any point in the system

Fractional Flow at Reservoir Converted to Surface Conditions

Assuming water as the displacing phase:

\[ f_w = q_w / q_t = q_w / (q_w + q_o) \]

Where:
- \( f_w \) = fraction of the total flow rate at a given point that is the displacing phase at reservoir conditions of temperature and pressure
Fractional Flow at Reservoir Converted to Surface Conditions

\[ W_c = \frac{B_o}{(B_w^* \left((1 - f_w) - 1\right) + B_o)} \]

Where:
- \( W_c \) = the amount of water produced at surface, as a fraction of the total production of oil and water
- \( B_o \) = reservoir volume of oil (at reservoir temperature and pressure) / surface volume of oil at standard conditions
- \( B_w \) = Reservoir volume of water / surface volume of water

Fractional Flow – Reservoir Conditions

\[ \frac{q_o \mu_o}{k_o} = -\left[ \frac{\partial p_o}{\partial u} + g\rho_o \sin \alpha \right] \]
\[ \frac{q_w \mu_w}{k_w} = -\left[ \frac{\partial p_w}{\partial u} + g\rho_w \sin \alpha \right] \]

Where:
- \( u \) = the linear direction of flow (measured from the inlet end)
- \( \partial p_o / \partial u \) = the pressure gradient in the oil phase
- \( \partial p_w / \partial u \) = the pressure gradient in the displacing phase
- \( \alpha \) = the angle of the fluid flow with respect to the horizontal
  ( updip flow is assigned to be "+"; downdip is "-"
- \( q_o \) = displaced fluid (oil) flow rate per unit of cross-sectional area normal to \( u \)
- \( q_w \) = displacing fluid flow rate per unit of cross-sectional area normal to \( u \)
- \( \rho \) = fluid density
- \( \mu \) = fluid viscosity
- \( o \) = subscript indicating displaced or oil phase, and
- \( w \) = subscript indicating displacing phase

See Reservoir Flow Properties Core for more information on Darcy's law.
Fractional Flow – Reservoir Conditions

- There are two simplifications to Darcy’s law to get the fractional flow equation of both oil and water.

**Thanks to:**

**Capillary pressure**
(Pressure difference across the interface of two fluids in a confined space)

and **Algebra**, you get:

\[
P_c = P_o - P_w
\]

\[
f_w = \frac{1 - 0.001127 k_o \left( \frac{\partial p_x}{\partial u} + 0.434 (\Delta \gamma) \sin \alpha \right)}{\mu_o q_t} + \frac{k_w \cdot \mu_w}{k_w \cdot \mu_o} \]

**Units:** md, cp, Bbl/ft², feet, psi; \((\Delta \gamma = \gamma_w - \gamma_o)\)

---

**Fractional Flow – Reservoir Conditions**

1. **Flow rate and angle of dip**
2. **Fluid viscosity and density**
3. **Capillary pressure**
   - Saturation, wettability, interfacial tension, pore structure
4. **Effective fluid permeabilities**
   - Absolute permeability, grain size, rock type
**Define Fluid Mobility**

\[ \lambda_w = \frac{k_w}{\mu_w} = k \frac{k_{rw}}{\mu_w} \]
\[ \lambda_o = \frac{k_o}{\mu_o} = k \frac{k_{ro}}{\mu_o} \]

Where:
- \( \lambda_w \) = mobility of the displacing phase (water), darcy/cp
- \( k_w \) = effective permeability of the displacing phase, darcies
- \( \mu_w \) = viscosity of the displacing phase, cp
- \( \lambda_o \) = mobility of the displaced phase (oil), darcy/cp
- \( k_o \) = effective permeability of the displaced phase, darcies
- \( \mu_o \) = viscosity of the displaced phase, cp

**Mobility Ratio**

Of greater importance is the mobility ratio – how the displacing or behind fluid moves with respect to the displaced or ahead fluid:

\[ M = \frac{(\lambda_w)_{behind}}{(\lambda_o)_{ahead}} = \frac{k_w (S_w) \mu_o}{k_o (S_{wi}) \mu_w} = \frac{k_{rw} (S_w) \mu_o}{k_{ro} (S_{wi}) \mu_w} \]

Where:
- \( k_{rw} (S_w) \) = the relative permeability of the displacing phase evaluated at the average displacing-phase saturation behind the front
- \( k_{ro} (S_{wi}) \) = relative permeability to oil evaluated at the initial water saturation in the oil bank
**Mobility Ratio**

Compares fluid mobilities at two points in the system

<table>
<thead>
<tr>
<th>Unfavorable displacement</th>
<th>Favorable displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>M &gt; 1</td>
<td>M &lt; 1</td>
</tr>
<tr>
<td>Displacing fluid moves more easily</td>
<td>Oil has greater mobility</td>
</tr>
</tbody>
</table>

**Fractional Flow Influences**

- Displacement efficiency is increased when angle of dip is increased
  - Water injected low on structure
  - Water injection rate is decreased, slower displacement is better
  - Density difference is greatest

In a waterflood:
Fractional Flow Influence on Wettability

- $K_w$ will be lower for a water-wet system
- Thus, the mobility term (denominator) will be lower

$$f_w = 1 - \frac{0.001127}{\mu_w q_t} \left[ \frac{\partial p_w}{\partial u} + 0.434 (\Delta y) \sin \alpha \right]$$

$$1 + \frac{k_o}{k_w} \cdot \frac{\mu_w}{\mu_o}$$

Water-wet reservoirs yield a higher displacement efficiency than comparable oil-wet reservoirs.

Fractional Flow Viscosity Influences on Mobility Term

Viscosity $\rightarrow$ Mobility Ratio

- Displacement efficiency is improved by:
  - Increasing water viscosity with polymers
  - Decreasing oil viscosity using hot water or steam
Gravity Term

Maximum displacement of oil will result when $f_w$ is kept to a minimum

Thus, you would like the Gravity term,

$$\frac{0.001127}{q_t\mu_o} \left[ \frac{\partial p_c}{\partial u} + 0.434 (\Delta \gamma) \sin \alpha \right]$$

to be as large as possible.

Maximum at $\sin 90^\circ$

Fractional Flow Influence on Gravity Term

$$f_w = 1 - \frac{0.001127}{\mu_o q_t} \left[ 1 + \frac{k_w}{k_o} + \frac{\mu_w}{\mu_o} \right] $$

$$f_w = \frac{1 - \frac{0.000488}{k_w A} (\Delta \gamma) \sin \alpha}{1 + \frac{k_w}{k_o} \frac{\mu_w}{\mu_o}}$$

Field Units:
- $k$ (md)
- $\mu$ (cp)
- $q_t$ (RB/D)
- $A$ (ft$^2$)
- $\alpha$ (degrees)
- $\Delta \gamma = (\gamma_{water} - \gamma_{oil})$

**Neglect $\frac{\partial p_c}{\partial u}$** since it is only significant at the front
- Front is region of rapid saturation change
- Neglecting derivative results in shock front
Neglecting capillary pressure and the effects of gravity:

\[ f_w = \frac{1}{1 + \frac{k_o \mu_w}{k_w \mu_o}} \]

This is the most common form of the fractional equation.

The resulting curve:
Problem 1 – Part 1

Open Problem A and calculate fractional flow curves for one set of rock curves, but three different oil viscosities. Assume the reservoir is horizontal, so you can use the form of the equation where the capillary pressure and gravity terms are ignored.

One fractional flow curve (For oil viscosity of 50 cp and a water viscosity of 0.5) has been calculated for you in column D (problem of the worksheet) and plotted in the curve. Calculate and plot the fractional flow curves for oil viscosities of 5 and 0.5 cp.

After you are done, review the solution document and see how your curves compare. Consider what these curves tell you about how oil viscosity affects displacement by water.

In the next section, we will consider what to do with the fractional flow curves to solve for the production of water and oil with time. That is, we are taking lab data and applying it to the displacement of oil in a real reservoir.
Why Calculate Fractional Flow Curves?

What is the purpose of calculating the fractional flow equations for a displacement process?

- Used to compute the saturation distribution in a linear waterflood system as a function of time
  - Saturation distribution is a function of the slope of the fractional flow curve
- Used to get the oil recovery behind the front at breakthrough
- Can be used to predict oil recovery and required water injection vs time
Shock Front Travels through Reservoir

- Each saturation plane travels at its own constant speed and is proportional to the gradient of the $f_w$ curve.
- The displacing fluid saturation moves at the same velocity at the leading edge of the front.

Efficiency of Displacement

Piston Versus Non-piston Displacement
Frontal Advance and the Welge Solution

Water Accumulation = Flow of Water In - Flow of Water Out

Water Saturation

Injection Distance Production

(Swirr - Sw) dSw/dt = QtΔfw

Mobility Ratio

Via substitution and integration:

x = (5.615 Wi / φA) * dfw / dSw

Where:

x (ft) = distance traveled by a fixed saturation. For a linear system, the front is traveling perpendicular to the cross-sectional area A (ft²), refer to the lavender Time bars on our displacement diagram.

Wi = cumulative water injected, bbls

t = time, days

dfw / dSw = derivative of the fractional flow curve at the saturation of interest, and must be equal for all saturations in the stabilized zone
**Welge Graphical Solution**

- Simple graphical solution at water breakthrough
  - Shock front saturation
  - Average saturation behind front
- Water saturation at flood front is determined from point of tangency on fractional flow curve (greatest slope) change (Part IV of Problem)
- Average water saturation behind the flood front is determined from where the tangent line intersects $f_w = 1.0$

---

### Problem A Parts II & III

- Calculate the $S_w$ at breakthrough and the average $S_w$ behind the front for each Reservoir in Part I

---

### Recovery Efficiency at Water Breakthrough

\[
E_D = \frac{\bar{S}_{wBT} - S_{wi}}{1 - S_{wi}} \quad \text{or} \quad E_D = \frac{S_{oi} - \bar{S}_{oBT}}{S_{oi}}
\]

**Where:**

- $E_D$ = displacement efficiency or fractional recovery of the total original oil in place, at pore level (fraction)
- $S_w$ = mean displacing-phase saturation behind the front at breakthrough (found by extending tangent of $f_w$ curve to $f_w = 1.0$), fraction
- $S_{wi}$ = initial (irreducible) displacing-phase saturation, fraction

---

### Problem A Part IV

- Using Part III, the average $S_w$, calculate the $E_D$ at breakthrough for each of the three reservoirs
Surface Water Cut

Convert to a surface water cut by converting reservoir volumes to surface volumes using:

\[
W_c = \frac{B_o}{(B_w \times (1/F_w) - 1)}
\]

Where:
- \( W_c \) = Surface fraction of water of total oil and water (\( q_w/(q_o + q_w) \))
- \( B_w \) = Reservoir volume of water/surface volume of water
- \( B_o \) = Reservoir volume of oil (at reservoir temperature and pressure)/surface volume of oil at standard conditions

Problem A Part V – Using Part III and reading the \( F_w \) at breakthrough, calculate the corresponding \( W_c \) for each oil that has a different \( B_o \).

Welge Solution Results

1. \( S_w \) provides the fractional flow at the front, \( f_w' \), which is used to calculate the surface water cut at breakthrough
2. \( S_o \) provides the average oil saturation behind the front which is used to calculate the pore displacement efficiency, \( E_D \)
3. Breakthrough time (days) can be estimated from \( E_p \), OOIP and the producing oil rate:

\[
t = \frac{OOIP \times E_D}{q_o}
\]

Problem A Parts VI and VII – Calculate mobility ratio and, using Part IV as well as the data on OOIP and \( q_o \), calculate the \( t \) (days) for each reservoir to breakthrough.

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Welge Solution after Breakthrough

1. Capillary trapped (residual) oil
2. Capillary trapped (connate) water

Distance through reservoir

Swe

Sw

Swf

SwBT

Swe

Sw

fw

0% 100%

Welge Procedure after Breakthrough

1. Construct tangents to fractional flow curve at increasing values of water saturation.
2. Average water saturation behind the front is determined from intersection of tangents with $f_w = 1.0$
3. Calculate recovery efficiency

$$R.E. = \frac{\overline{S}_i - S_{wi}}{1 - S_{wi}}$$

Where:

- $Q_i = $ cumulative water injected, pore volumes
Welge Solution Formulas

After breakthrough, $S_w = S_{we}$

Water saturation at the production well

$W_{id} = \frac{1}{dS_w^{ref}} \left( \frac{\bar{S}_w - S_{we}}{1 - f_{we}} \right) \frac{q_i t}{PV}$

Where:

$W_{id}$ = the Cumulative water injected in the displacement process in bbl or m$^3$

$PV$ = pore volumes in reservoir bbl or reservoir m$^3$

$q_i$ = the rate of injection of water in bbls per day or m$^3$ per day

$t$ = time in days

$N_{pd}$ = Cumulative oil produced in the displacement process in bbl or m$^3$

Welge Solution

- After breakthrough, an extended period of high water cut production takes place to recover remaining mobile oil. If a tangent line cannot be constructed, then a stabilized front or oil bank will not form.
Map of Reservoir

The reservoir being studied is a thin, shallow, under-saturated oil accumulation, it is under-going line drive dispersed flow water flood, as shown below

Cross Section (Part 1)

The LD-A reservoir consists of four patterns with dimensions as follows:
Understanding the Fractional Flow Equation (Part 2)

Set up a table in Problem B.xls as shown below. Fill it in using Table 1, Figure 2, and your knowledge of the Weige solution to the fractional flow equation.

Plot of Fractional Flow (Part 3)

Calculate the frontal saturation and the average saturation of water behind the front.
Oil Recovery at Breakthrough (Part 4a)

Use either of the equations below to calculate Part 4. You should also prove to yourself that they are the same.

\[ E_D = \frac{\overline{S}_{wBT} - \overline{S}_{wi}}{1 - \overline{S}_{wi}} \]

\[ E_D = \frac{\overline{S}_{oi} - \overline{S}_{oBT}}{\overline{S}_{oi}} \]

Using your units of choice, calculate oil recovery at breakthrough and time to breakthrough.

Water Injection and Front Maintenance (Part 4b)

Review the reservoir rate of water to maintain reservoir pressure and surface rate from the total field of 2000 bopd. How much is it?

2400 bwpd (381 m³wpd)

The fractional flow equation has to be calculated at reservoir conditions and the difference between oil reservoir volume and water reservoir volume is 1.2/1 (or 20%).

Refer to the problem statement Bo and Bw.
Water Drive Recovery Efficiency, in Real Reservoir

Water drive oil recovery efficiency (above $p_o$)

$$R.E. = E_D \times E_V \times E_A + \left(1 - E_D \times E_V \times E_A\right) \left(\frac{B_o - B_w}{B_o}\right)$$

Where:
- $RE$ = water drive recovery efficiency, percent
- $E_D$ = pore level recovery from displacement
- $E_V$ = recovery vertically
- $E_A$ = recovery areally
- $B_o$ = initial oil formation volume factor, rb/STB
- $B_w$ = reservoir volume of oil (at reservoir temperature and pressure)/surface volume of oil at standard conditions

- Typical range of recovery efficiency: under 40% to over 60%
- Statistical estimate for water drive yields an average recovery efficiency of 51%

Water Injection and Front Maintenance (Part 4c)

Calculate the fractional flow of water at the time of breakthrough from the curve on the left. What is it? 77%

After breakthrough, do you think the required water rate will go up or down?

Down, because of the 20% volume difference between oil and water.
Oil Recovery at Breakthrough (Part 4d, Part 5)

Finish the problem using the equation below for water injection rate after breakthrough to maintain reservoir pressure.

\[
\text{Water Component} = (0.77 \times 2000 \times 1.0) + \text{Oil Component}(0.23 \times 2000 \times 1.20) =
\]

To understand the Welge solution after breakthrough, review the calculations in Part 5, as well as the solution.
Learning Objectives

This section has covered the following learning objectives:

- Recognize immiscible fluid displacement and sources of data used for calculations
- Describe the assumptions and limitations of the Buckley-Leverett theory
- Describe the derivation of the dispersed fractional flow equation
- Recognize the impact of mobility, gravity, and capillary pressure on the dispersed fractional flow equation
- Identify how the Welge solution can be applied before, during, and after breakthrough
This section will cover the following learning objectives:

- Determine the rock, fluid, operational, and reservoir geometric factors affecting water influx ($W_e$) into your reservoir
- Calculate $W_e$ using different models
- Understand which descriptive parameters you should change so you can match the calculated $W_e$ to the observed reservoir pressure
Artesian Aquifers

Critical aspects of an aquifer:

1. Head it can exert on hydrocarbon reservoir
2. Continuity from point of recharge
3. The fact that they are recharged
   - Often, aquifers under the sea are recharged better
4. Connection they have at point of contact with hydrocarbon reservoir

Water Influx from Aquifers

**Why is it important to account for aquifer water encroachment?**

- Account for additional drive energy
- Match historical reservoir pressure (OOIP)
- Predict future pressure-production performance (rate vs. time)
- Estimate oil-water contact movement
Water Influx from Aquifers

Why is it difficult to account for aquifer water encroachment?

- Aquifer influx depends more on aquifer properties than hydrocarbon reservoir properties
  - Location of wells relative to aquifer and water movement
  - Aquifer size
  - Aquifer porosity and permeability

Water Influx Mechanism

1. Pressure drop in the reservoir
2. Water flows from aquifer to reservoir
3. Pressure gradient grows in the reservoir and aquifer pressure drops from depletion
4. Rock and water expansion in the aquifer balances aquifer voidage
5. Aquifer pressure at outer boundary
   - Initial until transient reaches boundary
   - Declines with continuing water influx
   - Pressure may be maintained if aquifer outcrops
Aquifer Expansion

Maximum water encroachment, $W_{e,aq}$

$$W_{e,aq} = (c_f + c_w) \cdot (PV_{aq}) \cdot (p_{iaq} - \overline{p}_{aq})$$

How long water encroachment takes is a function of:
- Aquifer size ($PV_{aq} =$ aquifer pore volume)
- Aquifer permeability
- Connection of reservoir and aquifer
**Water Influx Models**

1. **Pot Aquifer**
   - Small well connected aquifer, limited use

2. **Van Everdingen and Hurst**
   - Most complete and useful

3. **Fetkovich Semi-Steady State**
   - Good for late periods and limited aquifers, but doesn’t define the transient period

4. **Carter-Tracy**
   - Analytical model good for simulation

5. **Schilthuis Steady-State**
   - Almost never applicable

---

**Van Everdingen and Hurst**

**Characteristics**

- Transient and bounded aquifer flow
- Solution to hydraulic diffusivity equation
- Most comprehensive approach to water influx
- Requires superposition
- Defines aquifer characteristics
- Implemented in material balance programs, including MBAL
Behavior of a Large Aquifer

Transient moves out until boundaries reached, then PSS (pseudo-steady state) behavior

Reservoir/Aquifer Boundaries
van Everdingen Hurst Solution (Field Units)

\[ W_e = (1.119 \phi h c_e r_o^2 f) \Delta p Q_d \]

Where:
- \( W_e \) = cumulative water influx, reservoir bbls
- \( \phi \) = porosity, fraction
- \( h \) = effective aquifer thickness, ft
- \( c_e \) = total compressibility of the aquifer, \( c_w + c_r \) psi \(^{-1}\)
- \( r_o \) = radius of the hydrocarbon reservoir, ft
- \( f \) = fraction of the reservoir boundary exposed to the aquifer (0 < f < 1)
- \( \Delta p \) = pressure drop across the original reservoir/aquifer boundary, psi
- \( Q_d \) = dimensionless cumulative influx term
Dimensionless Influx, $Q_d$

Function of Two Dimensionless Terms

$$r_d = \frac{r_a}{r_o}$$

$$t_d = \frac{0.00633 \cdot k \cdot t}{\phi \cdot \mu_w \cdot c_e \cdot r_o^2}$$

Where:

$r_a$ = radius of the aquifer, ft or m
$r_o$ = radius of the original reservoir/aquifer interface, ft or m
$k$ = aquifer permeability, md
$t$ = time, days

Aquifer influx depends on the aquifer size, aquifer rock properties, water compressibility, the pressure drop the aquifer sees at the reservoir/aquifer interface, and how long that pressure drop has occurred.
**Dimensionless Influx, \( Q_d \ 5 \leq r_d \leq 10 \)**

**Calculation Procedure**

1. Estimate a dimensionless radius \( r_d \) from regional geology
2. Calculate a dimensionless time \( t_d \) based upon aquifer properties and real time
3. Determine dimensionless water influx, \( Q_d \) as a function of \( r_d \) and \( t_d \)
4. Using the observed reservoir \( \Delta \rho \), calculate water influx for the time period
Superposition

Superposition is comparable to waves coming in and adding to one another with time, in terms of a combined pressure drop.

Van Everdingen and Hurst equations are based on a constant pressure drop at reservoir-aquifer boundary. However, the rate of change in pressure drop in a reservoir varies significantly over the producing life. Therefore, you must use superposition with time to account for the combined effects of multiple pressure drops.

Here is how superposition is applied with time and average pressure drop.

\[
\frac{p_t}{p_i} = \frac{p_{t-1} + p_{t+1}}{2}
\]

\[
\frac{p_j}{p_i} = \frac{p_{j-1} + p_{j+1}}{2}
\]

\[
\frac{p_k}{p_i} = \frac{p_{k-1} + p_{k+1}}{2}
\]

\[
\frac{p_x}{p_i} = \frac{p_{x-1} + p_{x+1}}{2}
\]
Application of Superposition

If you want to calculate the $W'_C$ that has occurred after 24 months, you need to take into account everything that has occurred before then.

At 24 months:
- $\Delta p_1$ has been occurring since time = 0 mo., or for 24 months,
- $\Delta p_2$ has been occurring since time = 6 mo., or for 18 months,
- $\Delta p_3$ has been occurring since time = 12 mo., or for 12 months,
- $\Delta p_4$ has been occurring since time = 18 mo., or for 6 months

So the calculated $W'_C$ at any time must take into account all of the $\Delta p$'s that have happened before then, and for how long they have happened.

Application of Superposition

Water Influx Calculations

Let:

- $t_e = \frac{0.00633kt}{\phi \mu c_w r_o^2} = Et$ (E-constant)
- $W_e = (1.119\phi h c_w r_o^2)\Sigma (\Delta p Q_d) = C \Sigma (\Delta p Q_d)$ (C-constant)
- $r_a = r_d / r_o$

For example: $r_d = 10$, $E = 0.01 \text{ days}^{-1}$, $C = 150 \text{ rb/psi}$
### Application of Superposition Example

#### After 1 Year:

<table>
<thead>
<tr>
<th>Time, yrs</th>
<th>Time Active, yrs</th>
<th>( t_d )</th>
<th>( Q_d )</th>
<th>( W_{Mrb} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3.65</td>
<td>3.65</td>
<td>60.4</td>
</tr>
</tbody>
</table>

#### After 2 Years:

<table>
<thead>
<tr>
<th>Time, yrs</th>
<th>Time Active, yrs</th>
<th>( t_d )</th>
<th>( Q_d )</th>
<th>( W_{Mrb} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3.65</td>
<td>3.65</td>
<td>60.4</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3.01</td>
<td>3.01</td>
<td>102.5</td>
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#### After 3 Years:

<table>
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<th>Time, yrs</th>
<th>Time Active, yrs</th>
<th>( t_d )</th>
<th>( Q_d )</th>
<th>( W_{Mrb} )</th>
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<td>102.5</td>
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<td>3</td>
<td>7.10</td>
<td>7.10</td>
<td>142.1</td>
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When you have an aquifer attached to your oil reservoir and have run a material balance over the last ten years, you are comparing the pressures from the aquifer function to the material balance and adjusting them year by year.

Each year you will become more accurate, because aquifers often take some time to reveal themselves.

You will reach a point when the pressures are adjusted and the cumulative water influx from the aquifer function matches the required cumulative water influx energy term in the material balance equation.

At this point you will have now selected the proper values for the attached aquifer size and permeability.
**Pot Aquifer**

The Pot aquifer model will give the wrong answer in time. Note that, as opposed to the Van Everdingen and Hurst model where we developed dimensionless time as a critical function, this equation has no time. It says there is instantaneous communication with the oil reservoir. However, it will give you the right answer in a cumulative sense based on the pressure drop at the aquifer FWL connection.

It might be useful if the aquifer response is high (i.e., fed by fault or other high k connectivity). Otherwise, it’s impractical except to understand that water influx is a function of the compressibility of the total volume of the aquifer.

\[
W_e = (c_f + c_w) \cdot PV_{aq} \cdot \Delta p
\]

- Assumes instantaneous response (no time dimension):
  - Small, well connected, high pi aquifer
- Quick and simple; cold just add to material balance
- Calculates maximum water influx for given \( \Delta p \)
Fetkovich Finite Aquifer

Overview

- Used for finite aquifers
  - Neglects transient effects
- Does not require superposition
- Closely approximates van Everdingen and Hurst under pseudo-steady state conditions
- Based upon productivity index and pressure depletion concepts

Based on two equations:

**Productivity index for the aquifer**

\[
\frac{dW_e}{dt} = q_w = J(\bar{p}_a - \bar{p}_r)
\]

**Aquifer material balance**

\[
\bar{p}_a = -\left(\frac{p_i}{W_{ei}}\right)W_e + p_i
\]

Where:

- \(q_w\) = water influx rate from aquifer, rbbl/d
- \(J\) = productivity index for the aquifer, rbbl/d/psi
- \(\bar{p}_a\) = average aquifer pressure, psi
- \(\bar{p}_r\) = pressure at the aquifer-reservoir boundary, psi
- \(p_i\) = initial aquifer pressure, psi
- \(W_{ei}\) = encroachable water-in-place at \(p_i\), rbbls
- \(W_e\) = cumulative water influx, rbbls
Fetkovich Finite Aquifer

Solved over \( n \) repeated time increments:

\[
\Delta W_{e,n} = \frac{W_{ei}}{p_i} \left( \bar{p}_{a,n-1} - \bar{p}_{r,n} \right) \left[ 1 - \exp\left( -\frac{Jp_i \Delta t_n}{W_{ei}} \right) \right]
\]

Where:

\[
\bar{p}_{a,n-1} = p_i \left( 1 - \frac{W_e}{W_{ei}} \right)
\]

\[
\bar{p}_{r,n} = \frac{P_{r,n} + P_{r,n-1}}{2}
\]

\[
W_e = \sum \Delta W_{e,n}
\]

Fetkovich Method

Productivity Indices for Radial and Linear Aquifers

<table>
<thead>
<tr>
<th>Type of Outer Aquifer Boundary</th>
<th>Radial Flow</th>
<th>Linear Flow</th>
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<tr>
<td>Finite, No-Flow</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Finite, Constant Pressure</td>
<td></td>
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</tr>
</tbody>
</table>

\[
J = \frac{0.00708khf}{\mu \ln \left( \frac{r_a}{r_o} \right) - 0.75}
\]

\[
J = \frac{0.003381kwh}{\mu L}
\]

Where:

- \( k \) = absolute permeability, md
- \( w \) = width, ft
- \( L \) = length of linear aquifer, ft
- \( f \) = fraction of reservoir boundary exposed to aquifer \((0 < f < 1)\)
- \( r_a \) = radius of aquifer, ft
- \( r_o \) = radius of the original reservoir/aquifer interface, ft
- \( h \) = aquifer thickness, ft
Aquifers in Numerical Simulation

1. Analytical
   - Carter-Tracy (numerical approximation to van Everdingen and Hurst), Fetkovich, etc.
   - Coupled to edge or bottom of grid

2. Numerical
   - Aquifer represented with discrete grid cells
   - Each cell can have its own unique properties

Aquifers Summary

Van Everdingen and Hurst
\[ W_e = c \sum \left( \Delta p Q_a \right) \]
- Constant terminal pressure solution
- Most comprehensive approach
- Time-based (superposition principle)
- Applicable to transient, late transient and PSS flow
- Tedious calculation procedure

Fetkovich
\[ \frac{dW_e}{dt} = J(\bar{p}_a - \bar{p}_r) \]
- Assumes PSS relationship
- Not applicable for transient response
- Avoids superposition calculations

Pot Aquifer
\[ W_e = cP \bar{V}_w \Delta p \]
- Assumes instantaneous response (no time dimension)
- Quick and simple
- Calculates maximum influx

Carter-Tracy
- Constant terminal rate solution
- Applicable to transient, late transient and PSS flow
- Avoids extensive superposition calculations
- Often preferred to analytical
Key Learnings

1. Aquifer performance is predominantly determined by the aquifer properties: size, degree/manner of connection to reservoir and permeability.

2. The aquifer goes through the same transient flow into pseudo-steady state behavior that we see in oil or gas wells.

3. Aquifer behavior can be modeled by various means, with the van Everdingen and Hurst and Carter-Tracy models being the most technically correct.

4. Historical reservoir production and pressure data is used to allow the engineer to "identify" an aquifer description that makes the material balance calculations converge on a reservoir and aquifer description that honors field performance.

   - This description can then be used to project future performance, but the reliability of this forecast is limited if the aquifer has not yet reached pseudo-steady state.
Learning Objectives

This section has covered the following learning objectives:

- Determine the rock, fluid, operational, and reservoir geometric factors affecting water influx (W_e) into your reservoir
- Calculate W_e using different models
- Understand which descriptive parameters you should change so you can match the calculated W_e to the observed reservoir pressure
Learning Objectives

This section will cover the following learning objectives:

✓ Identify the theoretical differences between segregated and dispersed flow
✓ Describe how segregated flow is characterized
✓ Identify what causes segregated flow instability in oil displacement by water
✓ Identify the critical rate calculation for unstable displacement
✓ Identify the pore level displacement efficiency calculation for segregated flow
✓ Explain why coning occurs and how it can be avoided
In This Section

- Gravity
- Mobility Ratio
- Contact Tilt

- Stability of Displacement Processes
- Water Under-Running
- Maximum Pore Level Displacement Efficiency

- Special Segregated Flow Case - Coning
Gravity

- Saturations are vertically segregated based on fluid density
- Residual oil saturation behind the front
- Low rates of flooding
- Large fluid density difference
- Good vertical permeability
- Small thickness with small capillary transition zone

If the formation is relatively flat, you only need representative relative permeability curves and the viscosity of the fluids in order to calculate a fractional flow term ignoring gravity.

With or without the gravity terms in linear displacement, you will need to have some additional information, including the thickness of the formation and the width of the formation to predict recoveries.
Gravity

- You will also need to know the thickness of the transition zone, the density difference between the fluids, and the formation dip.
- In cases where the gravity term is highest, as shown here, you can expect segregated flow.
Mobility Ratio

Thin transition zone

Water

\[ S_w = 1 - S_o \]
\[ S_o = S_{or} \]

Oil

\[ S_w = S_{wi} \]
\[ S_o = S_{oi} \]

Mobility ratio based on endpoint saturations

\[ M = \frac{\left( k_{rw} @ S_{rel} \right) \mu_o}{\left( k_{ro} @ S_{rel} \right) \mu_w} \]
Contact Tilt

Contact tilt occurs for dipping reservoirs as a function of reservoir processing rate ($q_t$)

$$\tan \beta = \frac{G - (M - 1)}{G} \tan \alpha$$

$$M = \frac{k_w}{k_o} \frac{\mu_o}{\mu_w}$$

$$G = \frac{0.000488 k_i k_{res} s_{ro} A \Delta y \sin \alpha}{q_t \mu_o}$$
Stability of Displacement Processes

Unconditionally Stable
\[ M \leq 1 \]

Unstable
Viscous forces are stronger than gravity (density) forces
\[ G < M - 1 \]
\[ M > 1 \]
**Water Under-Running**

Here is a review of the equations for the gravity term (G), from which you can now calculate the critical rate for your particular reservoir.

Water under-running occurs if total reservoir production rate overcomes gravity force.

\[
G = 0.000488 \frac{k_a k_{rw} @ s_{orw} A \Delta \gamma \sin \alpha}{q_t \mu_w}
\]

\[
q_{crit} = 0.000488 \frac{k_a k_{rw} @ s_{orw} A \Delta \gamma \sin \alpha}{(M - 1) \mu_w}
\]

A similar concept (equation) applies to gas-oil systems. This is known as gas overriding.

Gas and its overriding condition will be covered in the *Reservoir Fluid Displacement Fundamentals* module within the reservoir engineering series.
Maximum Pore Level Displacement Efficiency

- This equation is referring to only the lab-derived fraction of pore level recoverable oil from \( k_r \) curves of total oil in pore space of the reservoir, since behind the front in segregated flow, the oil saturation is at the end point, \( S_{or} \).
- Two phases are present, but only one phase is flowing.

\[
E_D = \frac{S_{oi} - S_{or}}{S_{oi}}
\]

- Dispersed flow \( E_D \) at breakthrough depends on the average saturation behind the front at that time.
- Segregated flow \( E_D \) at breakthrough depends on the residual saturation of oil behind the front at that time.
**Wow! Nice Oil Well!**

- Here is a well that is capable of producing 1347 bopd
- It has no geologic barriers through the interval and it has a relatively high macro $k_v/k_h$

\[
Q_o = 0.00708 \times K \times Kro \times H \times (\text{Preservoir} - \text{Pbhf}) \left( \frac{\mu_o \times Bo}{\ln(re/rw) + s} \right)
\]

Where:
- $K$ (md) = 15
- $Kro$ = 0.9
- $H$ = 25 ft [7.6 m]
- Preservoir = 3500 psia [24 MPa]
- Pbhf = 500 psia [3.4 MPa]
- Oil Viscosity = 0.5
- Bo (RB/STB) = 1.4
- Skin = 0
- Drainage Radius = 500 ft [152 m]
- Wellbore Radius = 0.25 ft [0.08 m]

**Qo (STBOPD) = 1347 per well**

---

**Whoa, Not So Fast!**

**Criteria for Coning**

- Water coning into a vertical well is due to exceeding a critical drawdown to overcome gravity (see cone of water in the figure below)
- When the $\Delta P$ exceeds the value shown below under the geometric conditions indicated, water will cone into a well of relatively homogeneous reservoir character
- By staying below the critical rate (see animation on water_drive), it is possible to avoid coning
- However, this rate is, in the majority of cases, sub-economical

\[
\Delta P > 0.433 \times (\gamma_w - \gamma_o) \times h_c
\]
Bottom Water Drive Vertical Well - Coning

- Critical Rate $q_{DC}$ - maximum oil flow rate that will avoid a cone breakthrough into the well
- $q_{DC}$ is a function of:
  - Oil zone thickness, $h$
  - Perforation interval, $D$
  - FWL distance away
- For a vertical well, you can now write the $q_{DC}$

$$q_{DC} = 0.0246 \times k \times (\rho_w - \rho_o) \times (h^2 - D^2) \frac{\mu_o \times Bo \times [\ln(re/rb)]}{\ln(re/rb)}$$

How Much Can You Produce to Avoid Water?

$$Q_o = 0.0246 \times K_a \times K_{ro} \times H \times (\rho_w - \rho_o) \times (h^2 - D^2) \frac{\mu_o \times Bo \times (\ln(re/rw) + S)}{Bo \times (\ln(re/rw) + S)}$$

Where:
- $K$ (md) = 15
- $K_{ro}$ = 0.9
- $H$ = 25 ft [7.6 m]
- $D$ (perfs) = 0.5
- Density Oil = 0.35 psi/ft [7.9 kPa/m]
- Density Water = 0.45 psi/ft [10.2 kPa/m]
- Oil Viscosity = 0.5
- Bo (RB/STB) = 1.4
- Skin = 0
- Drainage Radius = 500 ft [152 m]
- Wellbore Radius = 0.25 ft [0.08 m]

$Q_o$ (STBOPD) = 4 per well
If You Exceed the Critical Drawdown an Upside Down Cone Forms Perpendicular to the Iso-Potential Lines

Vertical Well

Horizontal Well

\( x_e = \text{effective length} \)

With a vertical well the \( x_e \) is the distance you can perforate the well, and since you want to get away from water, you perforate as little as possible.

\( x_e = \text{length of horizontal} \)

If you had a horizontal well, you could place the well parallel to the FWL, and the drawdown would be spread over the longer \( x_e \).

When is Vertical Well Coning Impact Greatest?

Geology is super important when planning horizontals versus verticals.
When is Vertical Well Coning Impact Greatest?

1. The macro \( k/k_w \) is high, i.e., there is good communication to water, and there are no barriers to perforations
2. The oil is viscous or heavy (low API or high SG)
3. The effective distance to the water is low
4. Production rate is higher than the critical rate
5. Reservoir is oil wet
Learning Objectives

This section has covered the following learning objectives:

- Identify the theoretical differences between segregated and dispersed flow
- Describe how segregated flow is characterized
- Identify what causes segregated flow instability in oil displacement by water
- Identify the critical rate calculation for unstable displacement
- Identify the pore level displacement efficiency calculation for segregated flow
- Explain why coning occurs and how it can be avoided
Learning Objectives

This section will cover the following learning objectives:

- Recognize the techniques available to separate a reservoir into vertical layers
- Explain how Lorenz techniques can divide flow units by the pore structure that occurs
In This Section

- This section will consider the descriptions vertically through a reservoir.
- Often these changes can be important in restricting the upward movement of water.
- There are two methodologies available to describe the variation in vertical properties:
  - Dykstra-Parsons Coefficient
  - Lorenz Techniques
Permeability Variation ($V_{DP}$ Steps)

- Dykstra Parsons Coefficient of Variability $V_{DP}$
  
1. Arrange core samples in order of decreasing $k$ value
2. Prepare cumulative frequency plot on log-normal paper
   
   $k$ vs. % greater than
3. Draw a straight line through data points putting more weight on central points
4. Calculate $V$ from $k_{50}, k_{84.1}$

**Permeability Variation $V_{DP}$**

$V = \frac{k_{50} - k_{84.1}}{k_{50}}$
Permeability Variation $V_{DP}$

Where:

$V = \frac{\sigma}{\bar{x}}$

$\sigma$ = standard deviation

$x$ = mean value

$\bar{x}$ = geometric mean

$\rightarrow$ log-normal distribution: $\bar{x}$ = geometric mean 68.3% of values within mean +/-1 standard deviation

$k_{st,1} = \text{mean} + 1 \text{ standard deviation}$

Advantages

• Ability to compare across reservoirs worldwide for use as an analog

• Allows for easy elimination of outliers

Disadvantage

• Eliminating outliers may result in data 1 – 2 standard deviations away from the mean that are significant to flow
Lorenz Coefficient

\[
\frac{\text{Area } ABCA}{\text{Area } ACDA}
\]

Coefficient values range from 0 to 1 in order of increasing heterogeneity

- It is important to retain the \( h \) in both sides of the axis
  - \( kh \) is in Darcy’s Law, indicating cumulative flow capacity at pores
- Most often coupled with pore throat size
- Alternatively, \( \Phi \) multiplied by \( h \) in the X axis as in the volumetric formula and indicates storage capacity, or pore body size
- Therefore this is a plot of the variation in pore throat geometry
Sample Lorenz Plot – SPE Monograph 3

Lorenz Coefficient

\[
\frac{\text{Area } ABCA}{\text{Area } ACDA}
\]

Coefficient values range from 0 to 1 in order of increasing heterogeneity.

Increasing heterogeneity

Completely homogenous

Completely heterogeneous

Fig. 6.1 Flow capacity distribution, hypothetical reservoir.
Lorenz Plot Honoring Stratigraphy
Learning Objectives

This section has covered the following learning objectives:

- Recognize the techniques available to separate a reservoir into vertical layers
- Explain how Lorenz techniques can divide flow units by the pore structure that occurs

KEY LEARNINGS

- A fractional flow curve is calculated from the relative permeability curve, coupled with the reservoir oil and water viscosities.
- The shape of the fractional flow curve can be graphically analyzed to identify the efficiency of the displacement process before water breakthrough and after.
- The mobility ratio is the mobility of the displacing fluid (behind the front) / the mobility of the displaced fluid.
  - A mobility ratio of less than 1.0 is favorable and leads to piston like displacement and better performance in both technical and economic terms.
- While the fractional flow curve is usually calculated ignoring the effect of gravity, the gravity term (for non-horizontal flow) can be utilized to calculate the potential benefit of processing the reservoir at a slower rate.
### Applied Reservoir Engineering

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<th>Properties</th>
<th>Analysis</th>
<th>Management</th>
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<td>Reservoir Material Balance Core</td>
<td>Reserves and Resources Core</td>
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<tr>
<td>Reservoir Rock Properties Core</td>
<td>Reservoir Material Balance Fundamentals</td>
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<td>Decline Curve Analysis and Empirical Approaches Core</td>
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